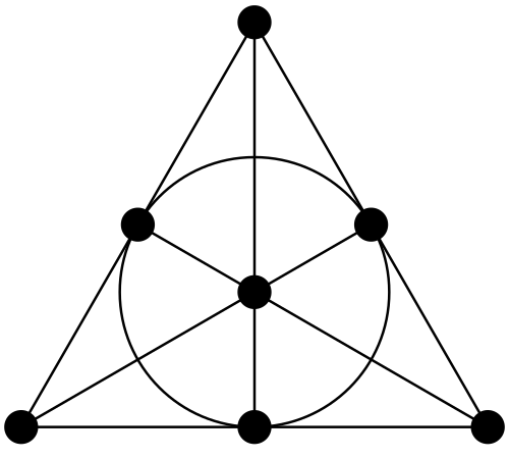
Week 6

Reading:

- *Geometry, Euclid and Beyond*: Chapter 2 Section 1

Exercises/questions:

1. The *Fano plane* is a finite set of points and lines as depicted below.



It is a plane, in the sense that it is built from points and lines. In particular, the lines are:

- The 3 sides of the equilateral triangle.
- The 3 altitudes of the equilaterial triangle.
- The inscribed circle.

The points are the vertices marked in the picture.

(a) Verify that the Fano plane satisfies Hilbert's axioms of incidence.

(b) What are the parallel lines of the Fano plane?

2. Suppose S is a finite set of points in the plane such that the area of triangle ABC is at most 1 whenever A, B, and C are in S. Show that there exists a triangle of area 4 that (together with its interior) covers the set S.

There are hints in the next few pages, a small hint on the next page, and a bigger hint after that. This is a hard problem, so just try to have fun with it.

Hint 1: Pick A, B, C in S so that $\triangle ABC$ has the largest possible area.

Hint 2: Pick A, B, C in S so that the area of $\triangle ABC$ is as large as possible, and use this triangle to make the construction from Corollary 5.2.